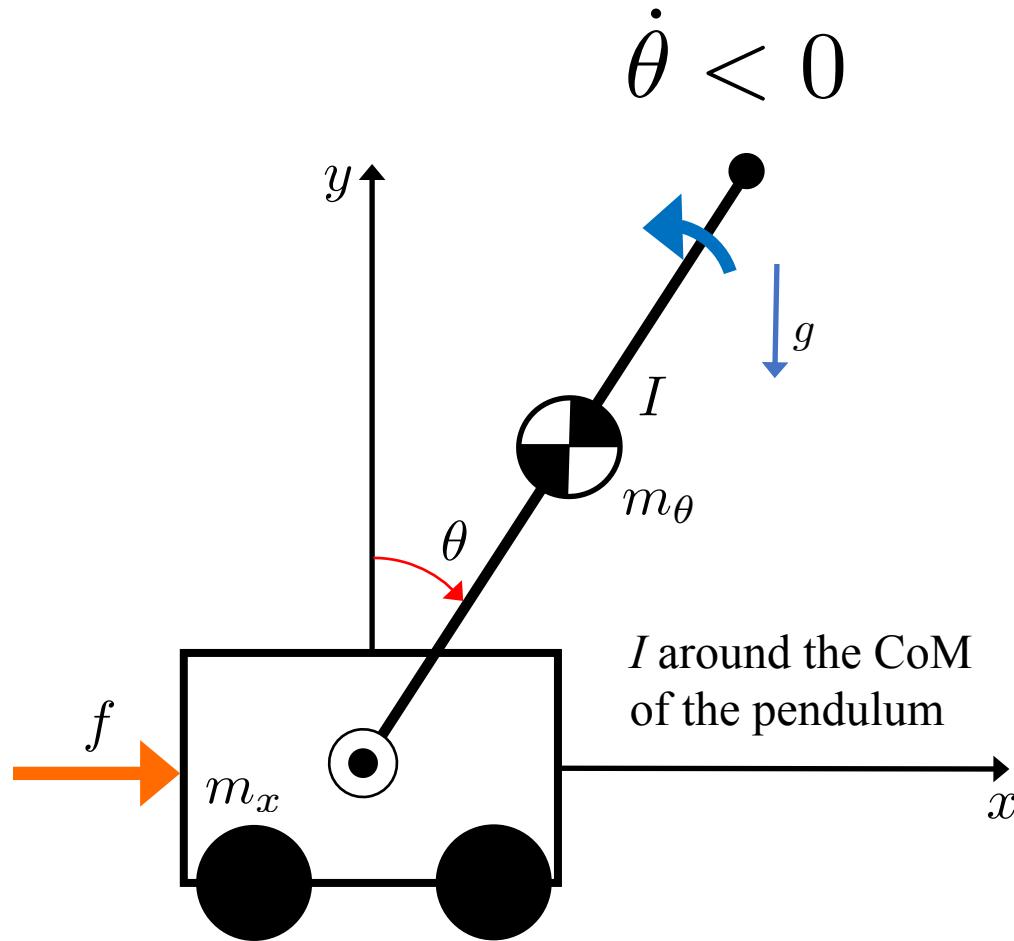


Revised Point for Today's Lecture

2019.1.11 Fri

Dai Owaki, Ph.D.

The Problem: Definition of Pendulum Angle



If we input a positive force, the CoM of pendulum moves the negative rotational direction of the angle



For stabilize the system, we put the following input

$$\begin{aligned}f &= -\{K_p(r - \theta) - K_v\dot{\theta}\} \\F(s) &= -\{K_p(R(s) - \Theta(s)) - sK_v\dot{\Theta}(s)\} \\&= -K_p(R(s) - \Theta(s)) + sK_v\dot{\Theta}(s)\end{aligned}$$

Why Is Python Correct?

Revised version of the python code

```
62 def Control(p):
63     x, dx, theta, dtheta = p
64
65     out = - K_p*(theta_d-theta) - K_v*(dtheta_d-dtheta)
66
67     return out
68
69 def InvertedPendulum(p, t):
70     x, dx, theta, dtheta = p
71
72     if theta > math.pi:
73         theta = -math.pi
74     elif theta < -math.pi:
75         theta = math.pi
76
77     M_11 = m_x + m_th
78     M_12 = m_th*l_g*math.cos(theta)
79     M_21 = m_th*l_g*math.cos(theta)
80     M_22 = I + m_th*l_g*l_g
81
82     #define matrix
83     M = np.matrix([[M_11, M_12], [M_21, M_22]])
84     N = np.matrix([[-m_th*l_g*math.sin(theta)*dtheta*dtheta], [0]])
85     G = np.matrix([[0], [-m_th*g*l_g*math.sin(theta)]])
86     F = np.matrix([[Control(p)], [0]])
87
88     IM = np.linalg.inv(M) # calc Inverse matrix
89     A = (-1)*IM.dot(N+G-F) # F is right hand side of equations
90
91     ddx, ddtheta = A
92
93     return [dx, ddx, dtheta, ddtheta]
```

$$\begin{aligned}f &= \boxed{-}\{K_p(r - \theta) - K_v\dot{\theta}\} \\F(s) &= -\{K_p(R(s) - \Theta(s)) - sK_v\dot{\Theta}(s)\} \\&= -K_p(R(s) - \Theta(s)) + sK_v\dot{\Theta}(s)\end{aligned}$$

$$\begin{aligned}\mathbf{F} &= [f, 0]^T, \boldsymbol{\theta} = [x, \theta]^T \\ \mathbf{F} &= \mathbf{M}(\boldsymbol{\theta})\ddot{\boldsymbol{\theta}} + \mathbf{h}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) + \mathbf{g}(\boldsymbol{\theta})\end{aligned}$$

$$\ddot{\boldsymbol{\theta}} = -\mathbf{M}(\boldsymbol{\theta})^{-1}\{\mathbf{h}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) + \mathbf{g}(\boldsymbol{\theta})\boxed{-}\mathbf{F}\}$$

I will upload the Stability Analysis for IP soon!!!